Anomalous temperature dependence of the Casimir force for thin metal films

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Within the framework of the Drude dispersive model, we predict an unusual non-monotonous temperature dependence of the Casimir force for thin metal films. For certain conditions, this force decreases with temperature due to the decrease of the metallic conductivity, whereas the force increases at high temperatures due to the increase of the thermal radiation pressure. We consider the attraction of a film to: either (i) a bulk ideal metal with a planar boundary, or (ii) a bulk metal sphere (lens). The experimental observation of the predicted non-monotonous temperature dependence of the Casimir force can put an end to the long-standing discussion on the role of the electron relaxation in the Casimir effect.

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The Casimir effect is one of the most interesting macroscopic manifestation of the zero-point vacuum oscillations of the quantum electromagnetic field. This effect manifests itself as the attractive force arising between two uncharged bodies placed in the vacuum due to the difference of the zero-point oscillation spectrum in the absence and in the presence of them (see, e.g., the monographs [1, 2] and review papers [3, 4]).

The Casimir effect attracts considerable attention because of its numerous applications in quantum field theory, atomic physics, condensed matter physics, gravitation and cosmology [1, 2, 3, 4, 5]. The noticeable progress in the measurements of the Casimir force [6] has opened the way for various potential applications in nanoscience [7], particularly, in the development of nanomechanical systems [2, 4, 7].

In spite of intensive studies on the Casimir effect, it is surprising that such an important problem as the temperature dependence of this effect is still unclear and is still an issue of lively discussion [8, 9, 10]. The central point in this discussion is if the Lifshitz formula (see, e.g., [11]) is applicable or not *for lossy media*. The authors of Ref. 8 have argued that the Drude dispersion relation for a lossy medium leads to inconsistencies because the reflection coefficient r_{TE} for the TE electromagnetic mode becomes discontinuous when the imaginary frequency $\zeta = -i\omega$ tends to zero. Therefore, instead of the Drude dispersion relation for the high-frequency dielectric permittivity ε,

$$\varepsilon(i\zeta) = 1 + \frac{\omega_p^2}{\zeta(\zeta+\nu)}, \qquad (1)$$

where ω_p and ν are the plasma frequency and the relaxation frequency, authors of Ref. 8 suggest the same equation, but with $\nu = 0$. Boström and Sernelius [9] have been the first to inquire whether this prescription is correct. They argued that in view of a realistic dispersion relation, the TE mode should not contribute to the Casimir force at zero temperature. Later, the authors of Refs. 9, 10 have shown that the mentioned discontinuity of r_{TE} at $\zeta \to 0$ does not lead to any physical difficulty or ambiguity.

For the case of zero temperature, the essence of the problem can be reduced to the following fundamental question: Can the Casimir force *"feel" the dissipation parameter* (the relaxation frequency ν) at zero temperature when *the dissipation itself is absent*? According to Ref. 8, the answer is "no". However, the authors of Refs. 9, 10 conclude that the answer should be "yes".

In this paper, we pay attention to an important feature of the Casimir force that can be demonstrated within the frame of the Drude dispersion model. There exist two competing phenomena that obviously determine the temperature dependence of the Casimir force. On the one hand, an increase in temperature leads to an increase of the relaxation frequency and, therefore, to a decrease of the metal conductivity and to a *decrease* of the Casimir force. On the other hand, when increasing the temperature, the Casimir force *increases* due to the growth of the thermal radiation pressure. The competition of these two effects can result in a *non-monotonous* temperature-dependence of the Casimir force.

The experimental observation of such an anomalous temperature dependence of the Casimir force might be a direct justification of the applicability of the Drude model. However, this temperature effect *for bulk metals* is very difficult to observe because of its small magnitude. Indeed, the relative contribution $|F_{\mathrm{rad}}/F(T=0)|$ of the thermal radiation force F_{rad} into the Casimir force

F is proportional to $(kTa/\hbar c)^4 \ll 1$ for realistic distances $a \ll a_T = \hbar c/kT$ between bulk metals. Here k is the Boltzman constant and c is the speed of light. The temperature-dependent part of the term F_ν, related to the relaxation frequency ν, is very small because it is proportional to the small surface impedance of a metal. Therefore, for bulk samples, the Casimir force is observed to be *slowly increasing* with T due to an increase of the radiation term $F_{\rm rad}$.

In this paper, we predict a *decreasing* Casimir force with T and show that the difficulties mentioned above, for the observation of the anomalous temperature dependence of the Casimir force, can be significantly diminished if we consider the interaction of *thin metal films*, instead of only between bulk samples. As was derived in Ref. 12, the temperature effects in the Casimir force can be brought to the forefront if the film thickness d is the smallest parameter with the dimension of length. The characteristic frequency ω_c of the fluctuations, that provide the main contribution to the Casimir force, becomes smaller,

$$\omega_c = \omega_p \sqrt{\frac{d}{a}} \ll \omega_p, \quad \omega_c \ll \frac{c}{a}, \tag{2}$$

if

$$d \ll \delta = c/\omega_p, \quad \delta \ll a. \tag{3}$$

This means that the high-temperature regime for the Casimir attraction of a film occurs at lower temperatures: at $T > T_c = \hbar\omega_c/k \propto d^{1/2}$. In addition, under conditions (3), the surface impedance of a metal film is not small. Therefore, the Casimir force for thin films becomes smaller than for bulk materials (see results of recent experiment [13] with thin films), and the relative role of the temperature effects in the Casimir force becomes stronger. Thus, as we demonstrate below, the anomalous temperature dependence of the Casimir force can be observed, in principle, for thin metal films. The successful implementation of this experiment could put an end to the longstanding discussion on the role of the electron relaxation in the Casimir effect.

Model.— The general formula for the Casimir interaction force between dielectric slabs with arbitrary dielectric constants ε was originally derived by Lifshitz [14] (see, also, Refs. 15, 16). The Casimir force is presented in this formula as a functional defined on the set of functions $\varepsilon(i\omega_n)$ of a discrete variable $\omega_n = 2\pi nkT$ ($n = 0, 1, 2, \ldots$). For the dielectric permittivity of the metal film, we choose the Drude dispersive model Eq. (1) which takes into account the temperature dependence of the relaxation frequency ν caused by the scattering of electrons by phonons. We use the relation,

$$\nu(T) = \nu_0 + \nu_{\rm ph}(T/\Theta),$$

$$\nu_{\rm ph}(x) = A\, \nu_{\rm ph}(1)\, x^5 \int_0^{1/x} \frac{y^5 dy}{({\rm e}^y - 1)\,(1 - {\rm e}^{-y})}, \tag{4}$$

based on the Grüneisen formula for the temperature dependence of the resistivity (see, e.g., Ref. 17). Here ν_0 is the residual relaxation frequency caused by the electron scattering on crystal defects, Θ is the Debye temperature, $\nu_{\rm ph}(T/\Theta)$ is the relaxation frequency due to the electron-phonon scattering. The value $\nu_{\rm ph}(1)$ depends on the Fermi velocity of electrons, the strength of the electron-phonon interaction, etc. This $\nu_{\rm ph}(1)$ can be be obtained by measuring the resistivity at the Debye temperature. The constant A is $\left(\int_0^1 y^5 dy/\left({\rm e}^y - 1\right)\left(1 - {\rm e}^{-y}\right)\right)^{-1} \approx 3$. For simplicity, we do not take into account the surface scattering of electrons in the explicit form (4) because it only changes the value of ν_0 (see, e.g., Ref. 18).

We consider first the Casimir effect for an ideal bulk conductor and a thin metal film of thickness d, separated by a distance a. Then, using the "Proximity Force Theorem" [19], we derive the expressions for the Casimir force between a metal film and an ideal metal sphere (lens). The geometry of problem is shown in Fig. 1.

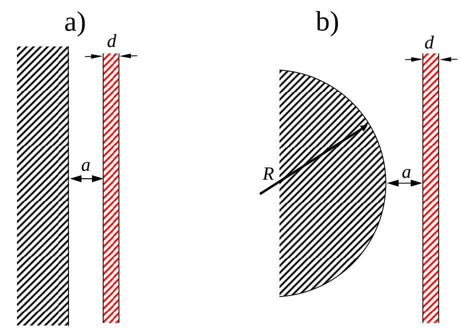

FIG. 1: (Color online) Geometry of the problem. (a) The Casimir attraction of a thin metal film (in red) to an ideal plane bulk metal. (b) The same film now interacting with a metal sphere.

Casimir force.— The asymptotic equation for the Casimir attraction of a *thin* metal film to an ideal *bulk* plane metal was derived in Ref. 12. The force f per unit area can be written in the form,

$$f = -\frac{BkT}{8\pi a^3} \int_0^\infty dx\, x^3 {\rm e}^{-x} I(x), \tag{5}$$

where $B = (\hbar\,\omega_c/4\pi kT)^2$, $\omega_c = \omega_p(d/a)^{1/2}$,

$$I(x) = {\sum_{n=0}^{\infty}}' \frac{1}{n(n+C) + B\Psi(x)}, \tag{6}$$

$\Psi(x) = x(1 - {\rm e}^{-x})$, $C = \hbar\,\nu(T)/2\pi kT$, the prime over the sum symbol indicates that the term with $n = 0$ is

taken with half the weight. Equation (5) is valid if conditions (3) are fulfilled. In this case, one can neglect the relativistic retarding effect and pass to the limit $c \to \infty$.

Using the Abel-Plana formula for summing series, we can rewrite Eq. (5) in the form of a sum of two terms that correspond to two sources for the temperature dependence of the Casimir force,

$$f = f_\nu + f_{\rm rad}, \tag{7}$$

$$f_\nu = -\frac{\hbar\omega_c}{32\pi^2 a^3}\int_0^\infty dx\, x^3 {\rm e}^{-x}\int_0^\infty \frac{d\tau}{\tau(\tau+\eta)+\Psi(x)}, \tag{8}$$

$$f_{\rm rad} = -\frac{\hbar\,\nu(T)}{8\pi^2 a^3}\int_0^\infty dx\, x^3 {\rm e}^{-x} \times \int_0^\infty \frac{tdt}{({\rm e}^{\gamma t}-1)\{[t^2-\Psi(x)]^2+\eta^2 t^2\}}, \tag{9}$$

where $\eta = 2\nu(T)/\omega_c$, $\gamma = \hbar\,\omega_c/2kT$.

The first term in Eq. (7) is provided by the quantum fluctuations of the electromagnetic field. It depends on temperature only via $\nu(T)$. Obviously, the modulus of this term *decreases* when increasing the temperature. The asymptotics of f_ν for low and high values of the parameter η are:

$$f_\nu = -\frac{i_1\hbar\,\omega_c}{64\pi a^3}\left(1 - i_2\frac{\nu(T)}{\omega_c}\right), \qquad \nu \ll \omega_c, \tag{10}$$

$$f_\nu = -\frac{3\hbar\,\omega_c^2}{16\pi^2 a^3\nu(T)}\left[\ln\left(\frac{2\nu(T)}{\omega_c}\right) - i_3\right], \ \nu \gg \omega_c, \tag{11}$$

where $i_1 \approx 3.51214$, $i_2 = 4\zeta(3)/\pi i_1 \approx 0.43578$, $i_3 \approx 0.59272$, and $\zeta(x)$ is the zeta function.

The second term in Eq. (7) is caused by the thermal fluctuations of the electromagnetic field. Its modulus *increases* when increasing the temperature. This term has different asymptotics in different temperature intervals:

$$f_{\rm rad}^{\rm low-T} = -\frac{\nu(T)(kT)^2}{24a^3\hbar\,\omega_c^2}\ln\left(\frac{\hbar\,\omega_c}{kT}\right), \ kT \ll \hbar\,\nu,\ \hbar\,\omega_c^2/\nu, \tag{12}$$

at low temperatures and

$$f_{\rm rad}^{\rm high-T} = -\frac{\zeta(3)}{8\pi}\frac{kT}{a^3}, \ kT \gg \min\left(\hbar\,\omega_c,\ \hbar\,\omega_c^2/\nu\right) \tag{13}$$

at high temperatures. In the case $\nu \ll \omega_c$, there exist the intermediate asymptotics,

$$f_{\rm rad}^{\rm intermed-T} = -\frac{\zeta(3)}{2\pi}\frac{(kT)^3}{a^3(\hbar\,\omega_c)^2}, \ \hbar\,\nu \ll kT \ll \hbar\,\omega_c. \tag{14}$$

Using Eqs. (10)–(14) and the Proximity Force Theorem [19], one can easily derive the analogue asymptotics for the Casimir force F,

$$F = 2\pi R\int_a^\infty da' f(a'), \tag{15}$$

between an ideal metallic sphere of radius R and a thin metal film:

$$F_\nu(\nu \ll \omega_c) = -\frac{i_1\hbar\,\omega_c R}{80a^2}\left(1 - \frac{5i_2}{4}\frac{\nu(T)}{\omega_c}\right), \tag{16}$$

$$F_\nu(\nu \gg \omega_c) = -\frac{\hbar\,\omega_c^2 R}{8\pi a^2\nu(T)}\left[\ln\left(\frac{2\nu}{\omega_c}\right) - i_3 + \frac{1}{6}\right]. \tag{17}$$

For the low-temperature interval in Eq. (12),

$$F_{\rm rad}^{\rm low-T} = -\frac{\pi R\nu(T)(kT)^2}{12a^2\hbar\,\omega_c^2}\left[\ln\left(\frac{\hbar\,\omega_c}{kT}\right) - \frac{1}{2}\right], \tag{18}$$

for the high-temperature interval in Eq. (13),

$$F_{\rm rad}^{\rm high-T} = -\frac{\zeta(3)}{8}\frac{RkT}{a^2}, \tag{19}$$

and for the intermediate temperature interval in Eq. (14),

$$F_{\rm rad}^{\rm intermed-T} = -\zeta(3)\frac{R(kT)^3}{a^2(\hbar\,\omega_c)^2}. \tag{20}$$

The above results show that the contribution to the Casimir force from quantum fluctuations always *decreases* when increasing the temperature. At the same time, the term related to the thermal radiation always *increases* with temperature. As a result of this competition, an interesting *non-monotonous* temperature dependence of the total Casimir force can be observed. Figure 2 shows the dependence of the Casimir force between a metal film and: (a) an ideal metal semi-space, and (b) a metal sphere (lens), on temperature, calculated for different values of the parameter $\nu_{\rm ph}(1)$. The Casimir force is normalized to the force $F_{\rm bulk}$ between ideal bulk metals for the same separation a. The force $F_{\rm bulk}$ is, $F_{\rm bulk}^{\rm plane} = \pi^2\hbar c\, S/240a^4$, for bulk samples with planar boundaries of the area S, and $F_{\rm bulk}^{\rm sphere} = \pi^3\hbar cR/360a^3$ for the attraction between a metal semi-space and a metal sphere with radius R. Note that we use the relations for $F_{\rm bulk}$ in the *retardation* limit because $c/\omega_p a \leq 1$. For the same separations a, the Casimir force for a thin film is calculated for the *non-retardation* regime, since $c/\omega_c a = c/\omega_p(da)^{1/2} \gg 1$.

The decreasing portion of the $F(T)$ dependence corresponds to a decrease of the Casimir contribution to the entropy when increasing T. This decrease is connected to the enhancement of the electron scattering on phonons and is much weaker than the increase of the phonon contribution to the entropy. Thus, the total entropy certainly increases when increasing the temperature.

In conclusion, we predict an unusual non-monotonous temperature dependence of the Casimir attraction force between a thin metal film and a bulk plane ideal metal or a metal sphere (lens). Usually, for bulk samples, the

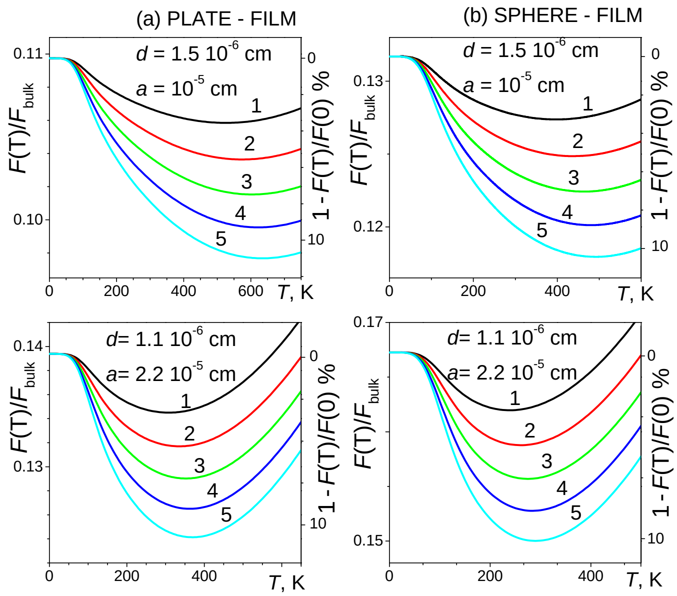

FIG. 2: The temperature dependence of the Casimir force of attraction of a metal film to an ideal plane bulk metal (a) and to a metal sphere (b) for different separations a. The left vertical axes show $F(T)/F_{\rm bulk}$ and the right vertical axes show $1 - F(T)/F(0)$. Curves 1, 2, 3, 4, 5 correspond to $\nu_{\rm ph} \cdot (10^{-13}\ {\rm s}) = 0.7$, 1.05, 1.4, 1.75, 2.1, respectively. The values of the other parameters are: $\omega_p = 2 \cdot 10^{15}\ {\rm s}^{-1}$, $\nu_0 = 10^{11}\ {\rm s}^{-1}$, $\Theta = 100$ K.

Casimir force *increases* slowly with temperature. Here we predict a *noticeable decrease* of the force with an increase of T for metal films. The experimental observation of this unusual temperature dependence of the Casimir force can put an end to the long-standing dispute on the role of the electron relaxation in the Casimir effect.

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